1.5 Exponential and Scientific Notation

1. Number Classification
2. Properties of Real Numbers
3. Use properties of exponents
4. Convert from scientific to decimal notation
5. Convert from decimal to scientific notation
6. Perform computations using scientific notation
7. Solve applied problems using scientific notation

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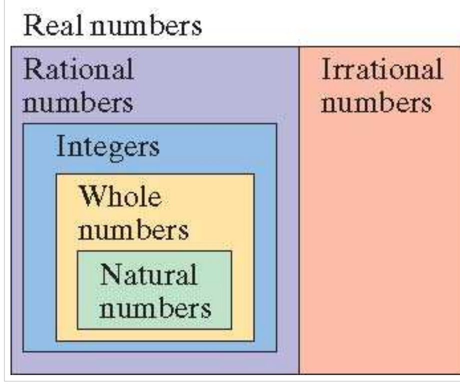
The Set of Real Numbers

$$\left\{-7,\ -\frac{3}{4},\ 0, 0.\overline{6},\ \sqrt{5},\ \pi,\ 7.3,\ \sqrt{81}\right\}$$



- Which of the numbers are:
  - Natural numbers?
  - Whole numbers?
  - Integers?
  - Rational numbers?
  - Irrational numbers?
  - Real numbers?

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Properties of the Real Numbers

	Property	Example
1.	Commutative Property of Addition $a + b = b + a$	$2 + 3 = 3 + 2$
2.	Commutative Property of Multiplication $a \bullet b = b \bullet a$	$2 \bullet (3) = 3 \bullet (2)$
3.	Associative Property of Addition $a + (b + c) = (a + b) + c$	$2 + (3 + 4) = (2 + 3) + 4$
4.	Associative Property of Multiplication $a \bullet (b \bullet c) = (a \bullet b) \bullet c$	$2 \bullet (3 \bullet 4) = (2 \bullet 3) \bullet 4$
5.	Distributive Property $a \bullet (b + c) = a \bullet b + a \bullet c$	$2 \bullet (3 + 4) = 2 \bullet 3 + 2 \bullet 4$
6.	Additive Identity Property $a + 0 = a$	$3 + 0 = 3$
7.	Multiplicative Identity Property $a \bullet 1 = a$	$3 \bullet 1 = 3$
8.	Multiplicative Inverse Property $a \cdot \left(\frac{1}{a}\right) = 1$ Note: a cannot $= 0$	$3 \cdot \left(\frac{1}{3}\right) = 1$

Properties of Exponents

Property	Meaning	Examples
Zero Exponent Rule $b^0 = 1$	If b is any real number other than 0 and exponent is zero the result is 1	$7^0 = 1$ $(-5)^0 = 1$ $-5^0 = -1$
The Product Rule $b^m \cdot b^n = b^{m+n}$	When multiplying exponential expressions with the same base, add the exponents.	$9^6 \cdot 9^{12} = 9^{6+12}$ $= 9^{18}$
The Power Rule $(b^m)^n = b^{mn}$	When an exponential expression is raised to a power, multiply the exponents.	$(3^4)^5 = 3^{4 \cdot 5} = 3^{20}$ $(5^3)^8 = 5^{3 \cdot 8} = 5^{24}$
The Quotient Rule $\frac{b^m}{b^n} = b^{m-n}$	When dividing exponential expressions with the same base, subtract the exponent in the denominator from the exponent in the numerator.	$\frac{5^{12}}{5^4} = 5^{12-4} = 5^8$ $\frac{9^{40}}{9^5} = 9^{40-5} = 9^{35}$

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The Negative Exponent Rule

- If b is any real number other than 0 and m is a natural number,

$$b^{-m}=\frac{1}{b^m}$$

$$8^{-2}=\frac{1}{8^2}=\frac{1}{8\cdot 8}=\frac{1}{64}$$

$$5^{-3}=\frac{1}{5^3}=\frac{1}{5\cdot 5\cdot 5}=\frac{1}{125}$$

$$7^{-1}=\frac{1}{7^1}=\frac{1}{7}$$

Exponent Exercises

- Negative Exponents causes switch between denominator and numerator

$t^{15}\cdot t^{-20}$ Prob 1.5.39

$\dfrac{3^5\cdot x^5}{3^7\cdot x^3}$ Prob 1.5.45

$\dfrac{(x^{-4}\cdot y^3)^5}{(x^7\cdot y^{-5})^{-2}}$ Prob 1.5.53

Powers of Ten

1. A positive exponent tells how many zeros follow the 1.
   For example, 10^9, is a 1 followed by 9 zeros: 1,000,000,000.
2. A negative exponent tells how many places there are to the right of the decimal point.
   For example, 10^{-9} has nine places to the right of the decimal point.

$$10^{-9} = 0.000000001$$

Scientific Notation

- A real number is written in scientific notation when it is expressed in the form

$M \times 10^n$ M = Mantissa, n = Exponent

  - where M is a number greater than or equal to 1 and less than 10 ($1 \le M < 10$), and n is an integer.
- Convert Scientific Notation to Decimal
  - If n is positive, move the decimal point in M to the right n places.
  - If n is negative, move the decimal point in M to the left $|n|$ places.

a. $2.6 \times 10^7 = 26{,}000{,}000$

$n = 7$

Move the decimal point 7 places to the right.

b. $1.1 \times 10^{-4} = 0.00011$

$n = -4$

Move the decimal point $|-4|$ places, or 4 places, to the left.

Convert Decimal to Scientific Notation

To write a real number in the form $M \times 10^n$:

- ❖ Determine M, the numerical factor. Move the decimal point such that $1 \le M < 10$
- ❖ Determine n, the exponent on 10^n. The number of places the decimal point was moved is n
  - ◆ The exponent n is positive if number $\ge \pm 10$
  - ◆ The exponent n negative if $\pm 1 >$ number > 0

a. $4{,}600{,}000 = 4.6 \times 10^6$

This number is greater than 10, so n is positive in $a \times 10^n$.

Move the decimal point in 4,600,000 to get $1 \le a < 10$.

The decimal point moved 6 places from 4,600,000 to 4.6.

b. $0.000023 = 2.3 \times 10^{-5}$

This number is less than 1, so n is negative in $a \times 10^n$.

Move the decimal point in 0.000023 to get $1 \le a < 10$.

The decimal point moved 5 places from 0.000023 to 2.3.

Scientific Notation Multiplication

❖ We use the product rule for exponents to multiply numbers in scientific notation:

$$(M \times 10^m) \times (N \times 10^n) = (M \times N) \times 10^{m+n}$$

- ◆ Add exponents on 10 and multiply Mantissas

Multiply: $(3.4 \times 10^9)(2 \times 10^{-5})$

$$(3.4 \times 10^9)(2 \times 10^{-5}) = (3.4 \times 2)(10^9 \times 10^{-5})$$

$$= 6.8 \times 10^{9+(-5)}$$

Answer in Scientific Notation

$$= 6.8 \times 10^4$$

$$= 68{,}000$$

Answer in Decimal Form

Scientific Notation Division

❖ We use the quotient rule for exponents to divide numbers in scientific notation

- ◆ Subtract exponents and divide Mantissas

$$\frac{M \times 10^m}{N \times 10^n} = \left(\frac{M}{N}\right) \times 10^{m-n}$$

Divide: $$\frac{8.4 \times 10^{-7}}{4 \times 10^{-4}} = \left(\frac{8.4}{4}\right) \times \left(\frac{10^{-7}}{10^{-4}}\right)$$ Regroup factors.

$$= 2.1 \times 10^{-7-(-4)}$$

Answer in Scientific Notation

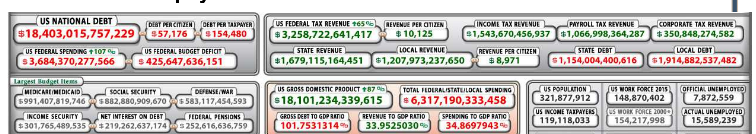
$$= 2.1 \times 10^{-3}$$ Subtract the exponents.

Answer in Decimal Form

$$= 0.0021$$

The National Debt Problem of the USA

As of October 10, 2015, the national debt was \$18.4 trillion, or 1.84×10^{13} dollars. At that time, the U.S. population was approximately 321,900,000, or 3.219×10^8. If the national debt was evenly divided among every individual in the United States, how much would each citizen have to pay?



Solution: The amount each citizen would have to pay is the total debt, 1.84×10^{13}, divided among the number of citizens, 3.219×10^{13}

$$\frac{1.84 \times 10^{13}}{3.219 \times 10^8} = \left(\frac{1.84}{3.219}\right) \times 10^{13-8} = 0.5716 \times 10^5 = 5.716 \times 10^4 = \$57{,}160$$

