5.4-6: Functions

- Graph quadratic functions.
  - Use quadratic models.
- Graph exponential functions.
  - Use exponential models.
- Graph logarithmic functions.
  - Use logarithmic models.
- Determine an appropriate function for modeling data.

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5.4: Modeling with Quadratic Functions

- A *Quadratic Function* is any function of the form

  $y = ax^2 + bx + c$ or $f(x) = ax^2 + bx + c$

  where a, b, and c are real numbers, with $a \neq 0$
- Quadratic functions graph as a *Parabola*
- The *Vertex* of the parabola is the lowest point *Minima* or the highest point *Maxima* on the graph

  Vertex is a point $=(V_x, V_y)$, where $V_x = \frac{-b}{2a}$ and $V_y = f(V_x)$
- y-intercept is constant coefficient c and is $(0, c)$
- x-intercepts may occur and can be found by solving for x when $f(x) = 0$, or $ax^2 + bx + c = 0$
  - Usually the Quadratic Formula is utilized to find the two solutions to the Quadratic Equation described above

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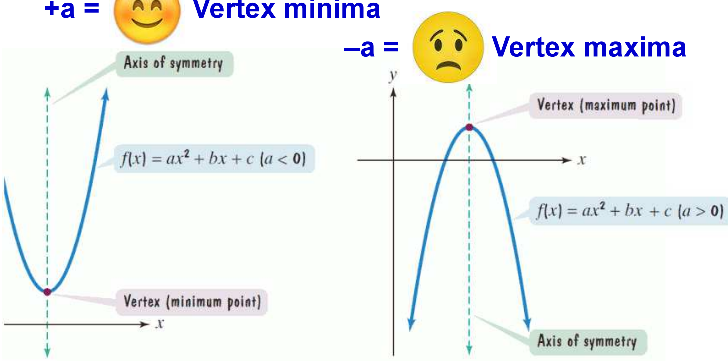
Modeling with Quadratic Functions

$$f(x) = y = ax^2 + bx + c$$

+a =  Vertex minima

–a = Vertex maxima



$a > 0$: Parabola opens upward. $a < 0$: Parabola opens downward.

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Graphing the Quadratic Equation

Graph the quadratic function: $f(x) = y = x^2 - 2x - 3$

a=+1 b=-2 c=-3

Solution: We follow the steps:

1) Determine how the parabola opens.
   Since a is the coefficient of x^2 and a = +1 in this case, then the parabola opens upward.

2) Find the vertex $V = (V_x, V_y)$
   Formula to find x-coordinate: $V_x = -\frac{b}{2a} = \frac{-(-2)}{2(1)} = 1$
   Plug x = 1 into the original function to find y-coordinate: $V_y = f(V_x) = (V_x)^2 - 2(V_x) - 3$
   $V_y = f(1) = (1)^2 - 2(1) - 3 = -4$
   The vertex V = (1,−4).

3) Find the y-intercept. $f(0) = y = 0^2 - 2(0) - 3 = -3$
   The parabola passes through the point (0,−3).

4) Find the x-intercepts. Solve for $f(x) = y = 0$, or $x^2 - 2x - 3 = 0$
   The x-intercepts are 3 and −1. $(x-3)(x+1) = 0$
   The parabola passes through (3,0) and (−1,0)

5) Plot the points and connect with parabola

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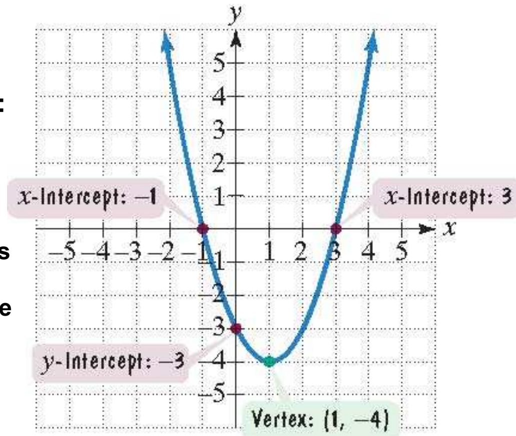
Plotting the Parabola

Plot the points:

1) Vertex
2) y-intercept
3) x-intercepts

Connect points with smooth Parabolic curve



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Earth Gravitation and Falling Objects

- An F-18 drops a bomb from an altitude of 8,000 feet above sea level on a target located at an elevation of 2,000 feet above sea level. The bomb altitude in feet after release is described by the following function A(t) as a function of t in seconds.

$$A(t) = -16t^2 + 8{,}000 \text{ feet}$$

  - What is the altitude of the bomb 10 seconds after release?
  - How many seconds will it take the bomb to reach its target?

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Ballistic Trajectory (y-axis only)

- A coastal defense canon fires a shell with an initial vertical velocity of 800 feet/second and an initial altitude of 200 feet above the water. The altitude of the shell can be approximated using the following function where A(*t*) is represents the altitude of the shell in feet at *t* seconds after launch:

$$A(t) = -16t^2 + 800t + 200 \text{ feet}$$

  - What is the altitude of the shell 30 seconds after launch?
  - What time does the shell reach its maximum altitude?
  - What is the maximum altitude of the shell?
  - At what time does the shell splash down in the water?

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5.5: Exponential Functions

- An Exponential function has the form:

$$y = f(x) = b^x$$

  - *b* is called the base and *b* > 0 and *b* ≠ 1
  - *x* can be any real number
- Euler's Constant is an irrational number that is frequently used as the base in an exponential function for mathematical models
  - e = 2.71828182846…
  - The number e is called the *natural base*.
- The *natural exponential function*.

$$y = g(x) = e^x$$

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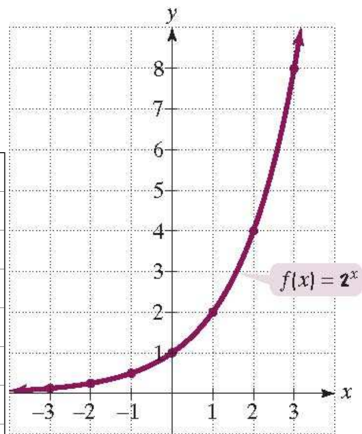
Graphing an Exponential Function

Graph: $f(x) = 2^x$

Solution: Start by selecting numbers for x and finding ordered pairs.
Graph the points in table

x	$f(x) = 2^x$	(x,y)
−3	$f(-3) = 2^{-3} = \frac{1}{8}$	$(-3,\frac{1}{8})$
−2	$f(-2) = 2^{-2} = \frac{1}{4}$	$(-2,\frac{1}{4})$
−1	$f(-1) = 2^{-1} = \frac{1}{2}$	$(-1,\frac{1}{2})$
0	$f(0) = 2^0 = 1$	(0,1)
1	$f(1) = 2^1 = 2$	(1,2)
2	$f(2) = 2^2 = 4$	(2,4)
3	$f(3) = 2^3 = 8$	(3,8)

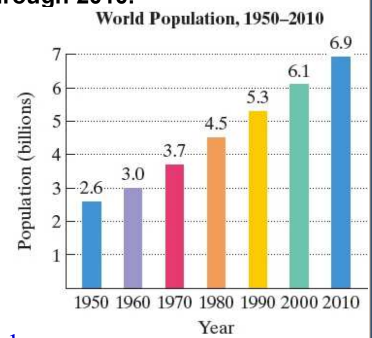


Exponential Models

- The bar graph below show the world population for seven selected years from 1950 through 2010.
- The $p(x)$ exponential function models world population in billions, x years after 1949.

$$p(x)=2.577(1.017)^x$$

- Use this model to determine the world population in 2000 and 2026



$$P(51) = 2.577(1.017)^{51} \approx 6.1$$

$$p(77) = 2.577(1.017)^{77} \approx 9.4$$

Alcohol and Risk of a Car Accident

- Medical research indicates that the risk of having a car accident increases exponentially as the concentration of alcohol in the blood increases. The risk is modeled by

$$R = 6e^{12.77x}$$

where x is the blood alcohol concentration and R, given as a percent, is the risk of having a car accident. In many states, it is illegal to drive with a blood alcohol concentration at 0.08 or greater. What is the risk of a car accident with a blood alcohol concentration at 0.08?

- **Solution:** We substitute 0.08 for x in the function.

$$R = 6e^{12.77x}$$

$$R = 6e^{12.77(0.08)}$$

- Putting this in the calculator, we get an approximation of 16.665813. Rounding to one decimal place, the risk of getting in a car accident is approximately 16.7% with a blood alcohol concentration at 0.08.

5.6: Logarithmic Functions

- Logarithmic functions have the form:

$$y=g(x)=\log_b x \qquad x=b^y$$

  - b is called the base and $b > 0$ and $b \neq 1$
  - $X > 0$ and can be any real number
- Standard Log function on calculator is base 10

$$y=g(x)=\log x=\log_{10} x \qquad x=10^y$$

- Natural Logarithmic functions have base e

$$y=h(x)=\ln x=\log_e x \qquad x=e^y$$

- Calculator determination of $\log_b x$

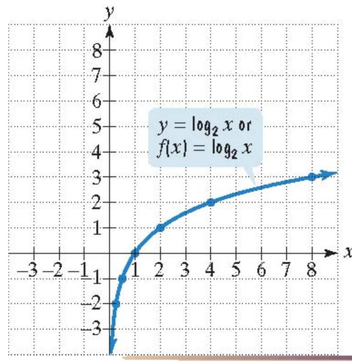
$$\log_b x=\frac{\log x}{\log b} \qquad \log_2 16=\frac{\log 16}{\log 2}=4 \qquad 16=2^4$$

Graphing a Logarithmic Function

Graph: $y = \log_2 x$

Solution: Because $y = \log_2 x$ means $2^y = x$, we will use the exponential equation to obtain the function's graph

$x = 2^y$	y	(x,y)
$2^{-2} = \frac{1}{4}$	-2	$(\frac{1}{4},-2)$
$2^{-1} = \frac{1}{2}$	-1	$(\frac{1}{2},-1)$
$2^0 = 1$	0	(1,0)
$2^1 = 2$	1	(2,1)
$2^2 = 4$	2	(4,2)
$2^3 = 8$	3	(8,3)



Temperature in an Enclosed Vehicle

When the outside air temperature is anywhere from 72° to 96°F, the temperature in an enclosed vehicle climbs by 43°in the first hour. The scatter plot is given below.

the function

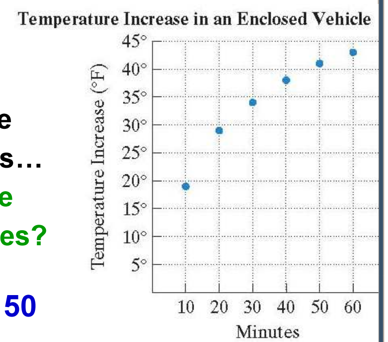
$f(x) = -11.6 + 13.4 \ln x$

models the temperature increase after x minutes…

What is the temperature Increase after 50 minutes?

$f(50) = -11.6 + 13.4 \ln 50$

$f(50) \approx 41$ degrees F



Earthquake Richter Log Scale

- An earthquake measured 63,100,000 times greater then the threshold intensity I_O, which is the weakest earthquake measurable on a seismograph. The magnitude on the Richter scale is defined by the function:

$$R(I) = \log\left(\frac{I}{I_O}\right)$$

- What is the Richter scale number of this earthquake?
- How does this compare with the 9.0 Tohoku Earthquake of 2011?

Determine Function for Modeling Data

Description of Data Points in a Scatter Plot	Model
Lie on or near a line	Linear function $y = mx + b$ or $f(x) = mx + b$
Increasing more and more rapidly	Exponential function $y = b^x$, or $f(x) = b^x$, $b > 1$
Increasing, although rate of increase is slowing down	Logarithmic function, $y = \log_b x$, $b > 1$ $y = \log_b x$ means $b^y = x$.
Decreasing and then increasing	Quadratic Function $y = f(x) = ax^2 + bx + c$ $a > 0$. The vertex is a minimum.
Increasing and then decreasing	Quadratic Function $y = f(x) = ax^2 + bx + c$ $a < 0$. The vertex is a maximum.