2.1: Algebraic Expressions

- *Algebra* uses letters, called *variables*, such as x and y, to represent numbers.
- *Algebraic expressions* are combinations of variables and numbers using the operations of addition, subtraction, multiplication, or division as well as exponents or radicals.
- Examples of algebraic expressions:

$c + 6$ $\quad 6y \quad x^2 - 6 \quad \sqrt{z} + 12$

Order of Operations Agreement = PEMDAS

1. Perform operations from within innermost grouping symbols to include **[{ ( ) }]** Horizontal Division bars are also considered grouping symbols separating a numerator group from a denominator group
2. Evaluate all exponential expressions
3. Perform multiplications and divisions as they occur, working from left to right
4. Perform additions and subtractions as they occur, working from left to right

Evaluating an Algebraic Expression

Evaluate: $7 + 5(x - 4)^3$ for $x = 6$

Substitute the value of x in the algebraic expression and simplify.

Solution:

$7 + 5(x - 4)^3 = 7 + 5(6 - 4)^3$ Replace x with 6

$= 7 + 5(2)^3$ Solve inside parentheses

$= 7 + 5(8)$ Evaluate exponent

$= 7 + 40$ Multiply

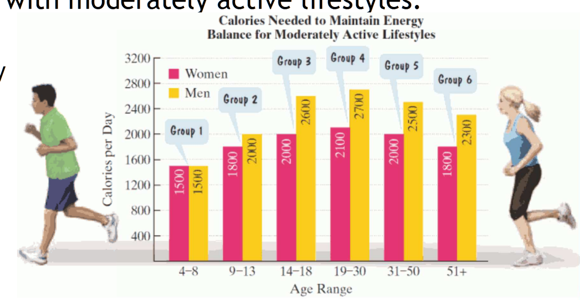
$= 47$ Add

Example: Modeling Caloric Needs

The bar graph shows the estimated number of calories per day needed to maintain energy balance for various gender and age groups for moderately active lifestyles.

The mathematical model $C = -66x^2 + 526x + 1030$ describes the number of calories needed per day by women in age group x with moderately active lifestyles.

According to the model, how many calories per day are needed by women between the ages of 19 and 30, inclusive, with this lifestyle?



Example Solution

- Because 19 through 30 is designated as group 4, we substitute 4 for x in the given model.

$$C = -66x^2 + 526x + 1030$$
$$= -66 \cdot 4^2 + 526 \cdot 4 + 1030$$
$$= -66 \cdot 16 + 2104 + 1030$$
$$= -1056 + 2104 + 1030$$
$$= 1048 + 1030$$
$$= 2078$$

- The formula indicates that 2078 calories are needed per day by women in the 19 through 30 age range with moderately active lifestyle.

2.2: Simplifying Algebraic Expressions

Use the Real Number Properties to simplify expressions

Commutative Property of Addition

$a + b = b + a$ $\qquad 13x^2 + 7x = 7x + 13x^2$

Commutative Property of Multiplication

$ab = ba$ $\qquad x \cdot 6 = 6 \cdot x$

Associative Property of Addition

$(a+b)+c = a+(b+c)$ $\qquad 3 + (8 + x) = (3 + 8) + x = 11 + x$

Associative Property of Multiplication

$(ab)c = a(bc)$ $\qquad -2(3x) = (-2 \cdot 3)x = -6x$

Distributive Property

$a(b + c) = ab + ac$ $\qquad 5(3x + 7) = 5 \cdot 3x + 5 \cdot 7 = 15x + 35$

$a(b - c) = ab - ac$ $\qquad 4(2x - 5) = 4 \cdot 2x - 4 \cdot 5 = 8x - 20$

Algebraic Expressions Terminology

- *Terms:* Those parts of an algebraic expression separated by addition or subtraction.
- Example: in the expression $7x - 9y - 3$
  - *Coefficient:* The numerical part of a term. $7, -9, -3$
  - *Constant:* A term that consists of just a number, also called a *constant term*. -3
  - *Like terms:* Terms that have the exact same variable factors and exponents. $7x$ and $3x$
  - *Factors:* Parts of each term that are multiplied $7x$, $-2 \cdot 3 \cdot 5$, $4 \cdot a \cdot c$
  - *Collecting like terms* utilizes distributive property $7x + 3 + 2x - 9y + 5 + 3y \quad \rightarrow 9x - 6y + 8$

Simplifying Algebraic Expressions

Simplify: $5(3x - 7) - 6x$

Solution:

$5(3x - 7) - 6x$

$= 5 \cdot 3x - 5 \cdot 7 - 6x$ distributive property

$= 15x - 35 - 6x$ multiply

$= (15x - 6x) - 35$ group like terms

$= 9x - 35$ combine like terms

Simplifying Algebraic Expressions

$12x^2y - 3xy^2 - 15x^2y + 10xy^2$ **Prob 2.2.29**

$15x - 12 - (4x + 9) - 8$ **Prob 2.2.39**

$(5x^2 - 3x - 9) - (x^2 - 5x - 9)$ **Prob 2.2.47**

$4 - 5[2(5x - 4^2) - (12x - 3^2)]$ **Prob 2.2.55**

2.3 Solving Linear Equations

- *Equation* is formed when an equal sign is placed between two algebraic expressions
- A *Linear Equation in one variable x* is an equation that can be written in the form

$$ax + b = 0$$

where a and b are real numbers, and $a \neq 0$

- *Solving an equation* in x involves determining all values of x that result in a true statement when substituted into the equation. Such values are *solutions*.
- *Equivalent equations* have the same solution set.

$4x + 12 = 0$ and $x = -3$ are equivalent equations.

Solving Using Properties of Equality

- The Addition Property of Equality

The same real number or algebraic expression may be added to both sides of an equation without changing the equation's solution set.

$a = b$ and $a + c = b + c$ are equivalent

$a = b$ and $a - c = b - c$ are equivalent

- The Multiplication Property of Equality

The same nonzero real number may multiply both sides of equation without changing the equation's solution set. $a = b$ and $a \cdot c = b \cdot c$ are equivalent

$a = b$ and $\frac{a}{c} = \frac{b}{c}$ are equivalent

Using Properties of Equality to Solve Equations

Equation	How to Isolate x	Solving the Equation	The Equation's Solution Set
$x - 3 = 8$	Add 3 to both sides.	$x - 3 + 3 = 8 + 3$ $x = 11$	$\{11\}$
$x + 7 = -15$	Subtract 7 from both sides.	$x + 7 - 7 = -15 - 7$ $x = -22$	$\{-22\}$
$6x = 30$	Divide both sides by 6 (or multiply both sides by $\frac{1}{6}$).	$\frac{6x}{6} = \frac{30}{6}$ $x = 5$	$\{5\}$
$\frac{x}{5} = 9$	Multiply both sides by 5.	$5 \cdot \frac{x}{5} = 5 \cdot 9$ $x = 45$	$\{45\}$

Solving a Linear Equation

1. Simplify the algebraic expression on each side by removing grouping symbols (apply distributive property) and combining like terms.
2. Collect all the variable terms on one side and all the constants, or numerical terms, on the other side.
3. Isolate the variable and solve.
4. Check the proposed solution in the original equation.

Example: $2(x - 4) - 5x = -5$

Step 1. Simplify the algebraic expression on each side

$2(x - 4) - 5x = -5$ This is the given equation

$2x - 8 - 5x = -5$ Use the distributive property

$-3x - 8 = -5$ Combine like terms: $2x - 5x = -3x$

Step 2. Collect variable terms on one side and constants on other side

$-3x - 8 + 8 = -5 + 8$ Add 8 to both sides and Simplify

$-3x = 3$

Step 3. Isolate the variable and solve

$\frac{-3x}{-3} = \frac{3}{-3}$ Divide both sides by 3 and Simplify

$\boldsymbol{x = -1}$ Solution

Step 4. Check the proposed solution in the original equation by substituting −1 for x

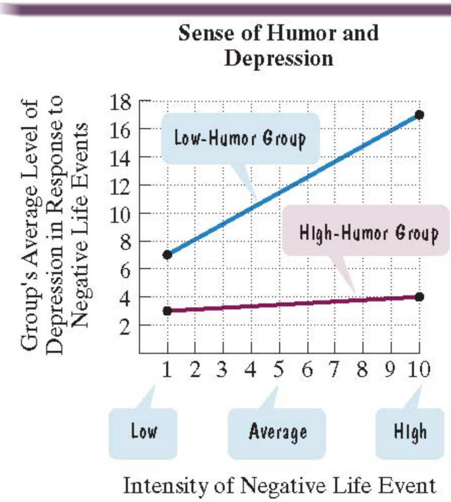
$2(x - 4) - 5x = -5$

$2(-1 - 4) - 5(-1) = -5$

$-10 - (-5) = -5$

$-5 = -5$ This statement is true

Application: Responding to Negative Life Events



These graphs indicate that persons with a low sense of humor have higher levels of depression. These graphs can be modeled by the following formulas:

Low Humor Group

$$D = \frac{10}{9}x + \frac{53}{9}$$

High Humor Group

$$D = \frac{1}{9}x + \frac{26}{9}$$

Alternate Solution: Clear fractions first

We are interested in the intensity of a negative life event with an average level of depression of 3 1/2 for the high humor group.

$$D = \frac{1}{9}x + \frac{26}{9}$$

Clear Fractions by multiplying both sides by $LCD = 9$

$$9 \cdot D = 9\left(\frac{1}{9}x + \frac{26}{9}\right)$$

$$9 \cdot D = x + 26$$

Substitute $\frac{7}{2}$ for D

$$\frac{9}{1} \cdot \frac{7}{2} = x + 26$$

$$63 = 2\,(x + 26)$$

Clear Fractions by multiplying both sides by of above by $LCD = 2$

$$63 = 2x + 52$$

$$63 - 52 = 2x + 52 - 52$$

$$11 = 2x$$

$$\frac{11}{2} = \frac{2x}{2}$$

$$\frac{11}{2} = x$$

$$x = \frac{11}{2}$$

Linear Equations with No Solution

- Solve: $2x + 6 = 2(x + 4)$
- *Solution:*

$$2x + 6 = 2(x + 4)$$
$$2x + 6 = 2x + 8$$
$$2x + 6 - 2x = 2x + 8 - 2x$$
$$6 = 8$$

- The original equation $2x + 6 = 2(x + 4)$ is equivalent to $6 = 8$, which is false for every value of x. The equation has no solution.
  The solution set is Ø, the empty set.

Linear Equations with Infinitely Many Solutions

- Solve: $4x + 6 = 6(x + 1) - 2x$
- *Solution:*

$$4x + 6 = 6(x + 1) - 2x$$
$$4x + 6 = 6x + 6 - 2x$$
$$4x + 6 = 4x + 6$$

- The original statement is equivalent to the statement $6 = 6$, which is true for every value of x. The solution set is the set of all real numbers, expressed as
  $\{x | x \text{ is a real number}\}$

Solving Linear Equations

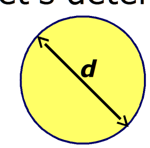
$4x - 3 = 13$	**Prob 2.3.19**
$7 - 2x = 3$	**Prob 2.3.23**
$-3(x - 5) = 6 - 4(2x - 1)$	**Prob 2.3.31**
$27 - 3(x + 4) = 4x - (2x - 20)$	**Prob 2.3.35**

2.4: Formulas = Literal Equations

- *Formula* is an equation that uses letters to express a relationship between two or more quantities represented by variables
- *Mathematical modeling* is the process of finding formulas to describe real-world phenomena

$$C = \pi \cdot d = \pi \cdot (2 \cdot r) = 2 \cdot \pi \cdot r$$

- Let's determine value of Pi experimentally.



$$\pi = \frac{C}{d}$$

Solving a Formula for One of its Variables

The total price of an article purchased on a monthly deferred payment plan is described by the following formula:

$$T = D + pm$$

T is the total price, D is the down payment, p is the monthly payment, and m is the number of months one pays.
Solve the formula for p.

Isolate p

$$T = D + pm$$

$$T - D = D - D + pm$$

$$T - D = pm$$

$$\frac{T - D}{m} = \frac{pm}{m}$$

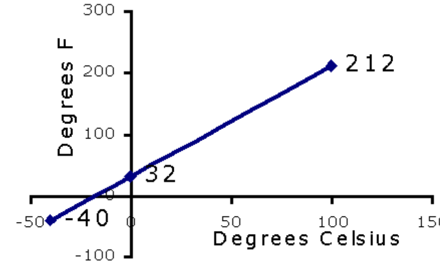
$$\frac{T - D}{m} = p$$

Algorithm Design - Mathematical

❖Mathematical Description

◆Boiling point
F =212
C = 100

◆Freezing point
F = 32
C = 0



$$y = mx + b$$

F = (180 / 100) C + 32
= (9/5) C + 32
= 1.8 C + 32

Solve the Formula for desired Variable

$P = 2L + 2W$, **Solve for W** Similar Prob 2.4.3

$F = C \cdot \frac{9}{5} + 32$, **Solve for C** Prob 2.4.13

$R_a = R_f \sqrt{1 - \left(\frac{v}{c}\right)^2}$, **Solve for $\frac{v}{c}$**

How long does it take to earn $1000

❖ Source Time Magazine: Who makes the highest wage? (Not self employed)