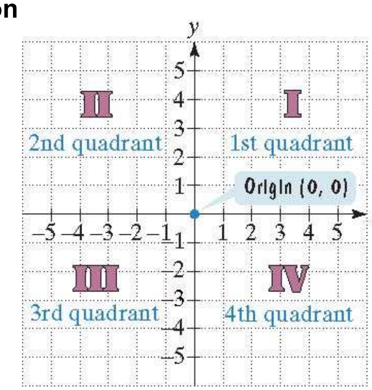
5.1: Graphs and Functions

- ❖Plot points in the rectangular coordinate system.
- ❖Graph equations in the rectangular coordinate system.
- ❖Use function notation.
- ❖Graph functions.
- ❖Use the vertical line test.
- ❖Obtain information about a function from its graph.

Points and Ordered Pairs

- ❖The horizontal number line is the x-axis.
- ❖The vertical number line is the y-axis.
- ❖The point of intersection of these axes is their zero point, called the origin.
- ❖Negative numbers are shown to the left of and below the origin.
- ❖The axes divide the plane into four quarters called "quadrants".

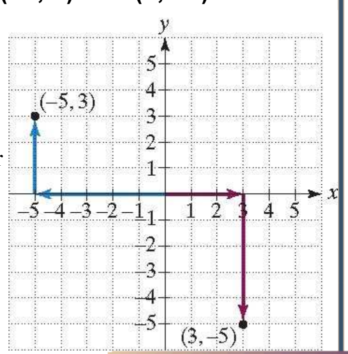


Points and Ordered Pairs

- ❖ Each point in the rectangular coordinate system corresponds to an ordered pair, (x, y).
- ❖ Look at the ordered pairs: (−5, 3) and (3, −5)
- ❖ The figure shows how we plot, or locate the points corresponding to the ordered pairs.

The first number in each pair, called the ***x*-coordinate**, denotes the distance and direction from the origin along the *x*-axis.

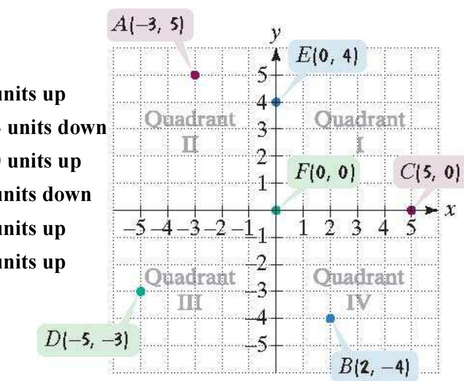
The second number in each pair, called the ***y*-coordinate**, denotes the vertical distance and direction along the *x*-axis or parallel to it.



Plotting Points in Coordinate System

- ❖ Plot points: A(−3, 5), B(2, −4), C(5,0), D(−5,−3), E(0, 4), F(0, 0).
- ❖ Solution: We move from the origin and plot the points as described below:

A(-3,5):	**3 units left, 5 units up**
B(2,-4):	**2 units right, 4 units down**
C(5,0):	**5 units right, 0 units up**
D(-5,-3):	**5 units left, 3 units down**
E(0,4):	**0 units left, 4 units up**
F(0,0):	**0 units left, 0 units up**



Graphs of Equations

- A relationship between two quantities can be expressed as an equation in two variables, such as

$$y = 4 - x^2$$

- A solution of an equation in two variables, x and y, is an ordered pair of real numbers with the following property:

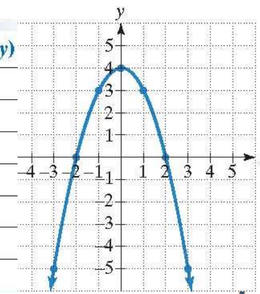
  When the x-coordinate is substituted for x and the y-coordinate is substituted for y in the equation, we obtain a true statement

- The graph of an equation in two variables is the set of all points whose coordinates satisfy the equation.

Graphing an Equation by Point-Plotting

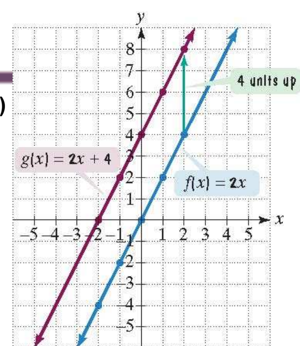
- Graph $y = 4 - x^2$ Select integers for x, from −3 to 3
- Solution: For each value of x, we find the value for y
- Now plot the seven points and join them with a smooth curve

x	$y = 4 - x^2$	Ordered Pair (x, y)
−3	$y = 4 - (-3)^2 = 4 - 9 = -5$	$(-3, -5)$
−2	$y = 4 - (-2)^2 = 4 - 4 = 0$	$(-2, 0)$
−1	$y = 4 - (-1)^2 = 4 - 1 = 3$	$(-1, 3)$
0	$y = 4 - 0^2 = 4 - 0 = 4$	$(0, 4)$
1	$y = 4 - 1^2 = 4 - 1 = 3$	$(1, 3)$
2	$y = 4 - 2^2 = 4 - 4 = 0$	$(2, 0)$
3	$y = 4 - 3^2 = 4 - 9 = -5$	$(3, -5)$



Graphing Functions

- If an equation in two variables (x and y) yields precisely one value of y for each value of x, y is a *function* of x.
- The notation $y = f(x)$ indicates that the variable y is a function of x. The notation $f(x)$ is read "f of x"
- Graph the functions $f(x) = 2x$ and $g(x) = 2x + 4$ in the same rectangular coordinate system. Select integers for x, $-2 \le x \le 2$



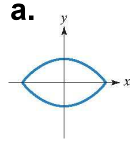
x	$f(x) = 2x$	(x, y) or $(x, f(x))$
−2	$f(-2) = 2(-2) = -4$	$(-2, -4)$
−1	$f(-1) = 2(-1) = -2$	$(-1, -2)$
0	$f(0) = 2 \cdot 0 = 0$	$(0, 0)$
1	$f(1) = 2 \cdot 1 = 2$	$(1, 2)$
2	$f(2) = 2 \cdot 2 = 4$	$(2, 4)$

x	$g(x) = 2x + 4$	(x, y) or $(x, g(x))$
−2	$g(-2) = 2(-2) + 4 = 0$	$(-2, 0)$
−1	$g(-1) = 2(-1) + 4 = 2$	$(-1, 2)$
0	$g(0) = 2 \cdot 0 + 4 = 4$	$(0, 4)$
1	$g(1) = 2 \cdot 1 + 4 = 6$	$(1, 6)$
2	$g(2) = 2 \cdot 2 + 4 = 8$	$(2, 8)$

Vertical Line Test

- If no vertical line intersects a graph more then once, then it is the graph of a function
- Every value of x determines one and only one value of $y = f(x)$
- Use the vertical line test to identify graphs in which y is a function of x

a.



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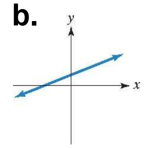

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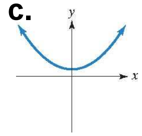

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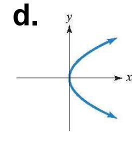

5.2: Graphing Linear Functions

- Use intercepts to graph a linear equation
- Calculate slope
- Use the slope and y-intercept to graph a line
- Graph horizontal and vertical lines
- Interpret slope as a rate of change
- Use slope and y-intercept to model data

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Using Intercepts to Graph a Linear Equation

- All linear equations in *parametric form* such as $Ax + By = C$ are straight lines when graphed
  - To locate *x-intercept*, set $y = 0$ and solve (?, 0)
  - To locate *y-intercept*, set $x = 0$ and solve (0, ?)
- Graph: $3x + 2y = 6$

Find x-intercept by letting y = 0 and solving for x.

$3x + 2y = 6$

$3x + 2 \cdot 0 = 6$

$3x = 6$

$x = 2$

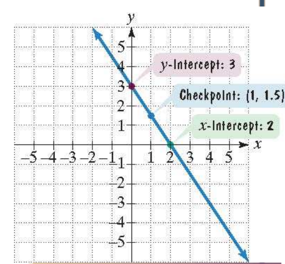
Find y-intercept by letting x = 0 and solving for y.

$3x + 2y = 6$

$3 \cdot 0 + 2y = 6$

$2y = 6$

$y = 3$



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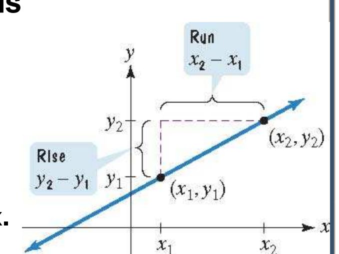
Slope of a Line

- Slope is defined as a *ratio of a change* in *y* to *x*
- Slope can be interpreted as a *rate of change* in an applied situation such as a word problem
- The *slope* of the line through the distinct points (x_1,y_1) and (x_2,y_2) is

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

where $x_2 - x_1 \neq 0$ change in x.



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Finding the Slope of a Line

The *slope* of the line through the distinct points (x_1,y_1) and (x_2,y_2)

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Find the slope of the line passing through the pair of points: (−3, −1) and (−2, 4).

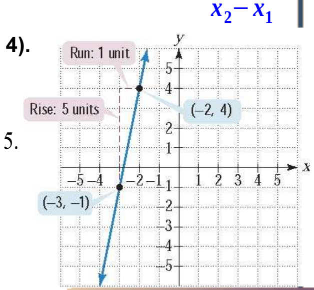
Solution:

Let $(x_1, y_1) = (-3, -1)$ and $(x_2, y_2) = (-2, 4)$.

We obtain the slope such that

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-1)}{-2 - (-3)} = \frac{5}{1} = 5.$$

Thus, the slope of the line is 5.



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Linear Equation: Slope-Intercept Form

- The slope-intercept form of the linear equation of a non-vertical line with slope *m* and *y*-intercept *b* is:

$$y = mx + b$$

- Graphing using the slope and y-intercept:
  1. Plot the point containing the *y*-intercept on the *y*-axis. This is the point (0,*b*).
  2. Obtain a second point using the slope *m*. Write *m* as a fraction, and use rise over run, starting at the point containing the *y*-intercept, to plot this point.
  3. Use a straightedge to draw a line through the two points. Draw arrowheads at the end of the line to show that the line continues indefinitely in both directions.

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Graphing Using Slope and *y*-intercept

- Graph the linear function by using the slope and *y*-intercept
- **Solution:** Since the graph is given in slope-intercept form we can easily find the slope and *y*-intercept.

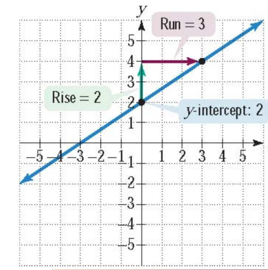
Slope y-int

$$y = \frac{2}{3}x + 2$$

Step 1 Plot the point containing the *y*-intercept on the *y*-axis. The *y*-intercept is (0, 2).

Step 2 Obtain a second point using the slope, *m*. We plot the second point at (3, 4).

Step 3 Use a straightedge to draw a line through the two points.



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Converting to Slope Intercept Form

- Graph the linear function $2x + 5y = 0$ by using the slope and *y*-intercept.
- **Solution:** We put the equation in slope-intercept form by solving for *y*.

parametric form $2x + 5y = 0$

$$2x - 2x + 5y = -2x + 0$$

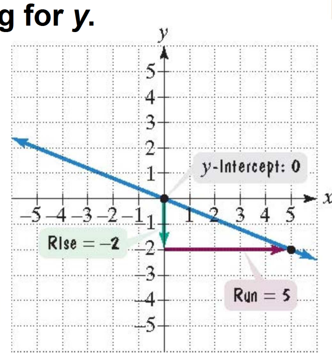
$$5y = -2x$$

$$\frac{5y}{5} = -\frac{2x}{5}$$

$$y = -\frac{2}{5}x$$

slope-intercept form $y = -\frac{2}{5}x + 0$

Slope y-int



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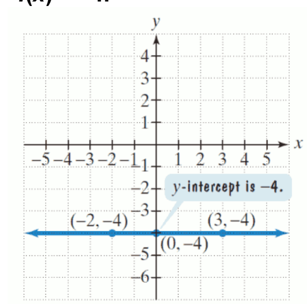
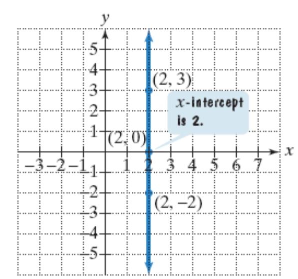
Horizontal and Vertical Lines

The graph of *y* = *b* or *f*(*x*) = *b* is a *horizontal line*. The *y*-intercept is *b*.

The graph of *y* = −4 or *f*(*x*) = −4.



The graph of *x* = *a* is a *vertical line*. The *x*-intercept is *a*.
The graph of *x* = 2.



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5.3: Systems of Linear Equations

- Two linear equations are called a *Linear System*
  - To solve for two unknowns you need two equations
  - A solution to a linear system is an ordered pair that satisfies both equations
- Determine whether (1,2) is a solution of the linear system:

  $2x - 3y = -4$

  $2x + y = 4$
- Solution: Because 1 is the x-coordinate and 2 is the y-coordinate of (1,2), we replace x with 1 and y with 2.

$2x - 3y = -4$	$2x + y = 4$
$2(1) - 3(2) = -4$	$2(1) + 2 = 4$
$2 - 6 = -4$	$2 + 2 = 4$
$-4 = -4$, TRUE	$4 = 4$, TRUE

- The pair (1,2) satisfies both equations; it makes each equation true. Thus, the pair is a solution of the system.

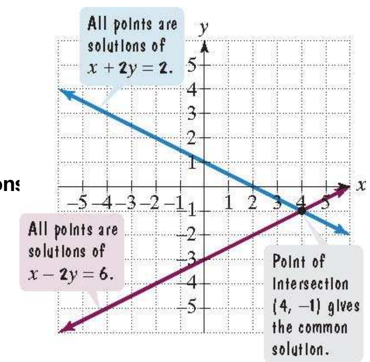
Solving Linear Systems by Graphing

- For a system with one solution, the coordinates of the point of intersection of the lines is the system's solution.
- Solve by graphing:

  $x + 2y = 2$

  $x - 2y = 6$
- Solution:

  Graph both lines in the same rectangular coordinate system.
  Use intercepts to graph equations
  x-intercept: Set *y* = 0:
  (2,0) (6,0)
  y-intercept: Set *x* = 0.
  (0,1) (0,−3)

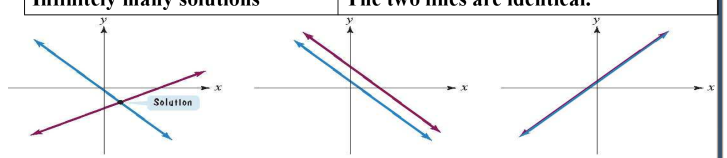


- We see the two graphs intersect at (4,−1). Hence, this is the solution to the system.

No Solution or Infinitely Many Solutions

The number of solutions to a system of two linear equations in two variables is given by one of the following:

Number of Solutions	What This Means Graphically
Exactly one ordered-pair solution	The two lines intersect at one point.
No Solution	The two lines are parallel.
Infinitely many solutions	The two lines are identical.



Exactly one solution — No solution (parallel lines) — Infinitely many solutions (lines coincide)

Solving Linear System by Substitution Method

Solve by the substitution method:

$x + y = -1$

$4x - 3y = 24$

Solution:

Step 1: Solve either of the equations for one variable in terms of the other. $y = -x - 1$

Step 2: Substitute the expression from step 1 into the other equation. $4x - 3(-x - 1) = 24$

Step 3: Solve the resulting equation containing one variable.

$4x + 3x + 3 = 24 \rightarrow 7x + 3 = 24 \rightarrow 7x = 21 \rightarrow x = 3$

Step 4: Back-substitute the $x = 3$ value into the equation from Step 1. $y = -3 - 1 = -4$ Therefore the solution is (3,−4).

Step 5: Check. Use this ordered pair to verify that this solution makes each equation true.

Solving Linear System by Addition Method

- Solve by the addition method:

$3x + 2y = 48$

Solution: $9x - 8y = -24$

Step 1: Rewrite both equations in the form $Ax + By = C$.
Both equations are already in this form.

Step 2: If necessary, multiply either equation or both equations by appropriate numbers so that the sum of the *x*-coefficients or the sum of the *y*-coefficients is 0.

$3x + 2y = 48 \xrightarrow{\text{Multiply by } -3} -9x - 6y = -144$

$9x - 8y = -24 \xrightarrow{\text{No Change}} 9x - 8y = -24$

Step 3 Add the equations. $-14y = -168$

Step 4 Solve the equation in one variable. $y = 12$

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Addition Method (continued)

Step 5: Back-substitute into one of the two equations and find the value for the other variable.

$3x + 2y = 48$

$3x + 2(12) = 48$

$3x + 24 = 48$

$3x = 24$

$x = 8$

Step 6: Check. The solution to the system is (8,12). We can check this by verifying that the solution is true for both equations.

$3 \cdot 8 + 2 \cdot 12 = 48$ TRUE

$9 \cdot 8 - 8 \cdot 12 = -24$ TRUE

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Linear System Exercise

- One pan pizza and two beef burritos provide 1980 calories. Two pan pizzas and one beef burrito provide 2670 calories. Find the caloric content of each item.

$p + 2b = 1980$

$2p + b = 2670$

Solution: Substitution method is used to evaluate.

Step 1: $p = 1980 - 2b$

Step 2: $2(1980 - 2b) + b = 2670$

Step 3: $3960 - 4b + b = 2670$

$1290 = 3b \rightarrow$ <u>$b = 430$ calories</u>

Step 4: $p = 1980 - 2(430) \rightarrow$ <u>$p = 1120$ calories</u>

Burrito = 430 calories, Pizza = 1120 calories

Step 5: Check $1120 + 2(430) = 1980$ TRUE

$2(1120) + 430 = 2670$ TRUE

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Linear System Exercises

- A hotel has 200 rooms. Those with kitchen facilities rent for \$100 per night and those without kitchen facilities rent for \$80 per night. On a night when the hotel was completely occupied, revenues were \$17,000. How many of each type of room does the hotel have?

  Kitchen Rooms = 50 rooms, Non-kitchen Rooms = 150 rooms

- Cholesterol intake should be limited to 300 mg or less each day. One serving of scrambled eggs from McDonald's and one Double Beef Whopper from Burger King exceed this intake by 241 mg. Two servings of scrambled eggs and three Double Beef Whoppers provide 1257 mg of cholesterol. Determine the cholesterol content in each item.

  McDonald's Scrambled Eggs = 366 mg cholesterol
  Burger King Double Beef Whopper = 175 mg cholesterol

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